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ABSTRACT

The effects of the violation of the assumption of normality coupled with the condition of multicollinearity upon the outcome of testing the hypothesis Beta equals zero in the two-predictor regression equation is investigated. A monte carlo approach was utilized in which three differenct distributions were sampled for two sample sizes over thirty-four population correlation matrices. The preliminary results indicate that the violation of the assumption of normality has significant effect upon the outcome of the hypothesis testing procedure. As was expected, however, the population correlation matrices with extremely high collinearity between the independent variables resulted in large standard errors in the sampling distributions of the standardized regression coefficients. Also, these same population correlation matrices revealed a larger probability of committing a type II error. Many researchers rely on beta weights to measure the importance of predictor variables in a regression equation. With the presence of multicollinearity, however, these estimates of population standardized regression weights will be subject to extreme fluctuation and should be interpreted with caution, especially when the sample size involved is relatively small. (Author/RC)

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The Effect of Multicollinearity and the Violation of the Assumption of Normality on the Testing of Hypotheses in Regression Analysis

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Abstract

This study investigated the effects of the violation of the assumption of normality coupled with the condition of multicollinearity upon the outcome of testing the hypothesis $\beta' = 0$ in the two-predictor regression equation. A monte carlo approach was utilized in which three different distributions were sampled for two sample sizes over thirty-four population correlation matrices. The preliminary results indicate that the combination of ~~neither~~ the violation of the assumption of normality with ~~nor the presence of multicollinearity~~ has no ~~any~~ significant effect upon the outcome of the hypothesis testing procedure. As was expected, however, the population correlation matrices with extremely high collinearity between the independent variables resulted in large standard errors in the sampling distributions of the standardized regression coefficients. Also, these same population correlation matrices revealed a larger probability of committing a type II error. Many researchers rely on beta weights to measure the importance of predictor variables in a regression equation. With the presence of multicollinearity, however, these estimates of population standardized regression weights will be subject to extreme fluctuation and should be interpreted with caution, especially when the sample size involved is relatively small.

The Effect of Multicollinearity and the Violation of the Assumption of Normality on the Testing of Hypotheses in Regression Analysis

One of the goals of applied research is to define functional relationships among variables of interest. If such relationships can be found, then this knowledge can be used for prediction purposes. For example given a subject's scores on selected X variables, the mathematical relationship can be utilized to predict that same subject's score on the associated Y variable. If the relationship is not a stable one, then perfect prediction is not possible. This is generally the situation that exists in social science research. The best that a prediction rule can do is to provide a 'good' fit to the data. Nevertheless, knowledge of such a rule can greatly decrease the errors in prediction and can be of practical utility in behavioral research (Hays, 1963).

Multiple linear regression is one mathematical approach to the problem of prediction. Given a set of independent variables and a criterion variable, least squares regression weights can be calculated which will maximize the squared multiple correlation between the criterion vector and the predicted criterion vector (Kerlinger, 1973). If the variables used in the determination of the regression weights are transformed into z score form, then the resulting weights are standardized regression coefficients and sometimes are referred to

as beta coefficients (McNemar, 1969). In the remainder of this paper the symbol β' will be used to refer to the population standardized regression coefficient and the symbol b' will represent the sample weight which estimates it.

These b' weights have been interpreted by some researchers to reflect the strength and direction of the relationship between an independent variable and the criterion. However, b' weights in most cases are not a useful measure of the importance of a predictor variable when the independent variables are highly intercorrelated (Darlington, 1968). There is no requirement in multiple regression analysis that the predictor variables used in the regression equation be uncorrelated or orthogonal (Johnston, 1963). From a linear algebra perspective this is reasonable since a criterion vector (dependent variable) can fit perfectly into a common vector space spanned by basis vectors (independent variables) which are not orthogonal. (The criterion vector can be a linear combination of these basis elements). Therefore, situations may occur in regression analysis in which the independent variables are highly intercorrelated. The presence of such highly intercorrelated predictors is termed multicollinearity. These predictor variables are, in fact, measuring approximately the same thing which makes the determination of the relative influence of each independent variable upon the criterion virtually impossible to disentangle (Goldberger, 1968). Also, the presence of multicollinearity increases the standard error of b' values which results in a statistically less consistent estimator of β' (Goldberger, 1968).

When exact multicollinearity occurs, one of the independent variables becomes a multiple of another. In the case of two predictor variables this would mean that the best fitting function which should be represented by a plane (see Figure 1) can instead be represented by a line. Again visualizing this situation from the perspective of linear algebra, it is evident that since linear dependencies cannot exist among basis elements which span a common vector space, the dimensionality of the vector space would in this case be reduced to two and the best fitting function would degenerate to one of a line. Exact multicollinearity is rare in applied research but multicollinearity is a rather common occurrance.

Statistical tests of significance can be run to determine whether or not a specific β' value is different from zero in the population. In order to test hypotheses such as these, an assumption of normality must be made in the distribution of the criterion measures (Draper & Smith, 1966). This assumption is rarely met in psychological or social science research. Many variables of interest to psychologists and educators are extremely skewed in the population making such an assumption invalid.

One of the goals of this study was to examine the effect of the violation of this assumption upon the probability of committing a type II error in the testing of hypotheses based upon b' coefficients. In order to answer this research question and the others which will be explained in turn, a monte carlo approach was taken. Extremely

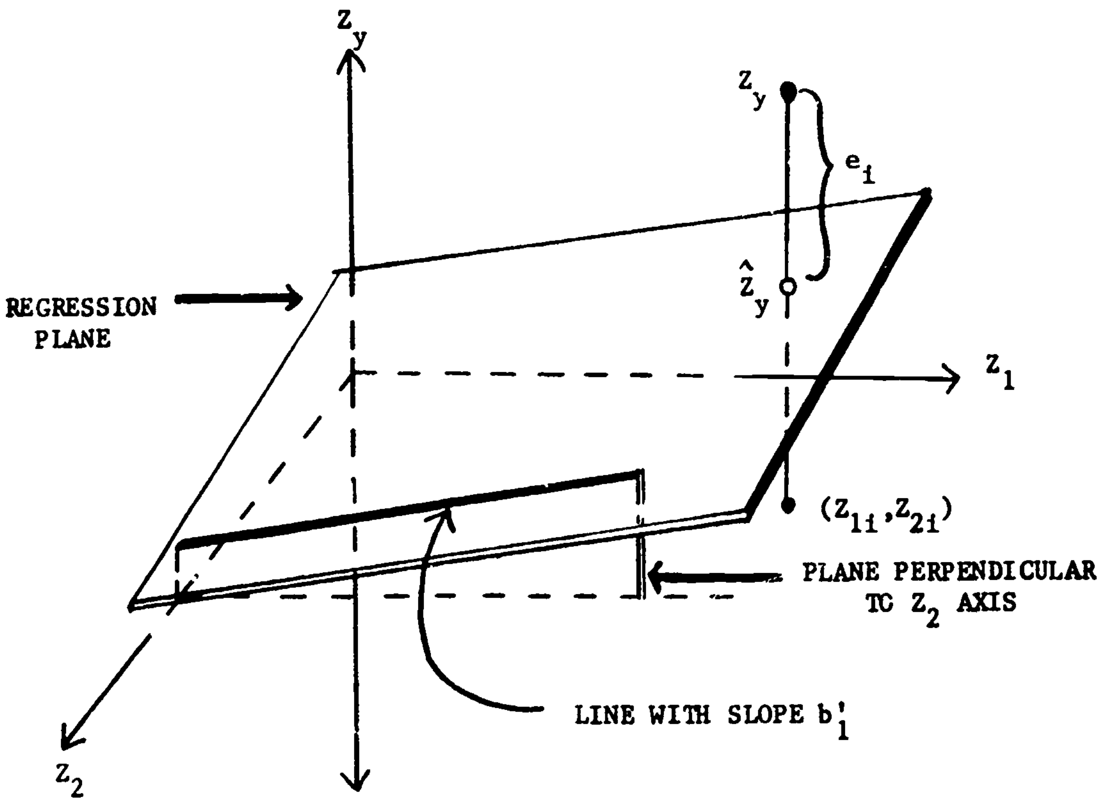
Z_y
Z_y
e_i
$\hat{Z}_y$
REGRESSION PLANE
Z_1
(Z_{1i}, Z_{2i})
PLANE PERPENDICULAR TO Z_2 AXIS
LINE WITH SLOPE b_1'
Z_2

Figure 1

skewed distributions were included in the distributions of the variables in the populations for the purpose of making the research more meaningful.

Turning once again to the problem of multicollinearity, one might consider the effect of highly correlated variables upon the outcome of the testing of hypotheses such as $H_0:\beta' = 0$ for each independent variable involved in the regression equation. Ostle (1963) states that the F tests used in testing these hypotheses are not all independent since the predictor variables themselves may be correlated. This was another goal of the study, to investigate the effects of multicollinearity upon the probability of committing a type II error in the testing of these hypotheses.

In review the main focus of the authors was the effect of multicollinearity coupled with the violation of the assumption of normality in the criterion measures upon the outcome of the testing of hypotheses concerning population regression coefficients in the two-predictor regression equation. Answers were sought to the following specific research question:

1. What effect does the violation of the assumption of normality have upon the probability of committing a type II error for alpha .05 in the testing of the null hypothesis $H_0:\beta_i' = 0$ $(i = 1,2)$ for both small and large sample sizes?
2. What effect does the presence of multicollinearity have upon the probability of committing a type II

error in the testing of these hypotheses for small and large samples?

3. Does this effect (if any) change as the distribution sampled becomes more skewed?

The mathematical model under investigation may be written as:

$$Z_y = \beta_1' z_1 + \beta_2' z_2 + e$$

or equivalently:

$$Z_y = Z_x \beta'' + e$$

where Z_y is an (n x 1) vector of observations in z score form

Z_x is an (n x 2) matrix of known form whose elements are also standardized

β'' is a (2 x 1) vector of parameters

e is an (n x 1) vector of errors

and where the e_i are independently and normally distributed (Draper & Smith, 1966). This last statement is needed in order to test the significance of β'. We must also make the important assumption that the linear model defines the best functional fit to the data in the population. This assumption can be met by sampling from a multivariate normal distribution (Blalock, 1972) which was accomplished through the monte carlo program.

The test of the null hypothesis that a specific β' value was different from zero was determined from the following test statistic (McNemar, 1969):

$$F = \frac{(R_1^2 - R_2^2)/(m_1 - m_2)}{(1 - R_1^2)/(N - m_1 - 1)}$$

where R_1 is the multiple correlation coefficient based upon m_1 of the predictor variables and R_2 is the multiple correlation coefficient based upon m_2 of the remaining variables where $m_2 = m_1 - 1$. Sample b' values were calculated using the following formulae (McNemar, 1969):

$$b'_1 = \frac{r_{y1} - r_{y2}r_{12}}{(1 - r_{12}^2)} \qquad b'_2 = \frac{r_{y2} - r_{y1}r_{12}}{(1 - r_{12}^2)}$$

The population correlation matrices, sample sizes and population distributions chosen will be outlined in the next section.

Method

In order to answer the research questions it seemed necessary to construct approximate sampling distributions of b'_1 and b'_2 values from the sample regression equation:

$$z_y = b'_1 z_1 + b'_2 z_2 + e$$

The hypotheses dealt with the violation of the assumption of normality, level of collinearity between the independent variables, sample size and the effect of these upon the hypothesis testing of β'. Three different distributions were chosen from which to generate random samples of z scores; the multivariate normal, χ^2 with 5 degrees of freedom and χ^2 with 20 degrees of freedom. Three different levels of intercorrelation between the predictor variables were chosen: ρ_{12} = .95, .70 and .45. In addition two different sample sizes were selected: n = 25 and n = 100.

The basic element in the monte carlo procedure was the intercorrelation between the independent variables in the population. At one level of intercorrelation between Z_1 and Z_2 different levels of correlation between Z_y and Z_1 were selected as were different levels of correlation between Z_y and Z_2. Thirty-four different triplets of population intercorrelations among Z_y, Z_1 and Z_2 were selected and are displayed in Table 1. Five cases involved a ρ_{12} value of .95, fourteen cases involved a ρ_{12} value of .70 and fifteen cases involved a ρ_{12} value of .45. These triplets of population Pearson Product-Moment correlation coefficients were transformed into factor structure matrices which were then used as input into a monte carlo program written by the main author and based upon a previously developed Frotran program (Wherry, 1965). By focusing in on one of the population correlation matrices, the logic behind the monte carlo technique can be more easily explained and comprehended.

For one set of fixed ρ_{y1}, ρ_{y2} and ρ_{12} values a factor structure matrix was calculated and a distribution and sample size were chosen for generating sample r_{y1}, r_{y2} and r_{12} values. Because the authors were interested in examining standardized regression coefficients which are based upon z score values, these sample r coefficients were all that was needed in order to calculate b_1' and b_2' coefficients for a sample regression equation. Five-hundred sample correlation matrices were produced for each selected distribution and sample size, therefore five-hundred sample regression equations in z score form were developed

for the population regression equation. The five-hundred b_1' coefficients were then used to form an approximate sampling distribution for b_1'. The same procedure was followed for b_2'.

As each sample b' value was produced, an F test was used to determine if the regression weight was significantly different from zero at the .05 level of significance. This information was tabulated and used in the calculation of the empirical probability of committing a type II error: which was estimated by taking the proportion of b' values which were retained in the hypothesis testing procedure. All the population β' values present in this study (see Table 1) were different from zero. Therefore, the only kind of error which could be examined was type II error; the probability of retaining a false hypothesis.

For each factor structure matrix six approximate sampling distributions for b_1' were developed and six approximate sampling distributions for b_2' were simultaneously developed. One was formed for each combination of distribution and n size: multivariate normal, χ^2_5 and χ^2_{20}; n=100 and n=25. Since there were thirty-four factor structures in total, two-hundred and four approximate sampling distributions were formed for each b' coefficient.

Characteristics of the sampling distributions, population ρ values, distributional type, sample size and population β' values were examined for the presence of relationships in accordance with the research hypotheses. Table 2 through Table 7a contain the summary statistics of the sampling distributions of each b'.

Results

Table 2 and Table 3 consist of calculations based upon the bias involved in each sampling distribution. Since the mod ved in the regression procedure was fixed, the mean of each sampling distribution of b' should equal the population β' value. In Table 2 and Table 3, however, there is evidence of bias. The average bias, whether mean or median, is slight: the ~~minimum~~ largest negative bias is -.056 while the ~~maximum~~ largest positive bias is .051. Since each sampling distribution involved a finite number of b' values and was, therefore, only approximate, it would seem logical to attribute the presence of bias to the approximation technique. By scanning each table across distributional shape, (Dist. Type), there appears to be little difference in the reported statistics and no consistent pattern appears as the deviation from normality becomes more marked. A Spearman correlation coefficient was calculated between bias and distribution shape and was found to be non significant in all cases. (see Table 8). Likewise, by scanning the columns of Table 2 and Table 3 there appears to be little difference in the reported statistics. A Spearman correlation coefficient was calculated between bias and level of intercorrelation between predictors in the population. This coefficient was also found to be nonsignificant in all but one case. (see Table 8).

Tables 4 and 5 contain statistics on the standard deviations of the sampling distributions of the b' values. Scanning across each table from left to right there appears to be little change in the average of the standard errors for the b' coefficients. The Spearman

correlation coefficient calculated between empirical standard error, (S_{e_1} and S_{e_2}), and distributional type was not found to be significant. As was expected, however, there is a significant correlation between the standard error of each b' sampling distribution and the level of intercorrelation present between the independent variables in the population. (see Table 8). By examination of Tables 4 and 5 one can see a decrease in the average standard error of the sampling distributions of the b' values as the ρ_{12} value decreases from .95 to .45. This decrease is consistent for a sample size of 25 and a sample size of 100 regardless of the distribution sampled. As the ρ_{12} value decreases, the spread of the standard error values for the distributions also decreases as indicated by the standard deviation statistics. ~~Also, the average standard error of the sampling distributions within ρ_{12} levels is smaller for an n size of 100.~~

In Table 6 and Table 6a there appear statistics calculated on difference values obtained by subtracting the theoretical probability of committing a type II error from the empirical proportion of false hypotheses which were retained. Again, there seems to be little change among the average of the difference values as the shape of the distributions sampled becomes more skewed. However, as ρ_{12} decreases, the average difference between empirical and theoretical probability of committing a type II error also decreases for n = 25. The maximum difference appears when ρ_{12} equals .95; the maximum difference at this level is .502. As ρ_{12} decreases to .45, the maximum difference is found to be .192. The spread of the difference values decreases as the ρ_{12} value

for n=25

decreases from .95 to .45. A significant correlation was found to exist between the difference values, ($Diff_1(25)$, $Diff_2(25)$), and ρ_{12} for a sample size of 25. As the sample size increased to 100, the correlation was found to be non-significant. These difference values can be attributed to the approximation of the monte carlo technique. When ρ_{12} was relatively low and the sample size was large, the approximation technique was much more accurate.

Table 7 and Table 7a contain the proportion of times the null hypothesis was falsely retained; an approximation of the probability of committing a type II error. As the distribution becomes more skewed, there is no significant change in the average proportion of times a false hypothesis was retained regardless of sample size. The Spearman correlation coefficient calculated between empirical proportion of type II errors committed and distributional shape was found to be non-significant regardless of sample size. The largest Spearman found was .03.

from .95 to .70 or from .95 to .45

As the ρ_{12} value decreases, the probability of committing a type II error also decreases as would be expected. This finding is consistent for all distributions sampled for both sample sizes. The average type II error ~~for sample sizes of 100~~ within a level of ρ_{12} is smaller for a sample size of 100 than for one of 25.

Conclusions and Implications

The results illustrate that a departure from normality in the distribution from which random samples are selected for inclusion in a regression equation with two predictors does not significantly influence the probability of committing a type II error in the testing of the null hypothesis $H_0:\beta_i' = 0$; $(i = 1,2)$. Because the assumption of normality can rarely be met in the distribution of psychological and educational variables, and if it seems plausible to generalize beyond two independent variables, the results indicate that this violation should not be of great concern to a researcher.

Level of intercorrelation confounded with a departure from normality did not significantly influence the probability of committing a type II error either.

As was expected, multicollinearity does have an effect upon the sampling distribution of b' values. This fact is consistent with the theory behind the effects of multicollinearity upon distributions of standardized regression coefficients. The more highly the predictor variables are correlated, the larger the standard error of the b' values. This implies that a confidence interval around a b' value for the purpose of estimating β' would have to be much larger in the case of a regression equation with an r_{12} value which is exceedingly high. In general The smaller the amount of collinearity between two predictors and the larger the sample size, the more statistically consistent the b' values are: in other words the probability that the b' value is close to the β' value of the population regression equation is increased.

Based upon the findings of this research report it would seem that researchers dealing with variables selected from populations with extremely skewed distributions do not have to be concerned with any detrimental effects upon the probability of committing a type II error. However, with small sample sizes and highly correlated predictors, generalizations about the contribution of an independent variable to any regression equation should be made with caution. Sample b' values in situations such as these are subject to extreme fluctuation and, although they are unbiased in the long run, most researchers are dealing with only one regression equation and, therefore, only one estimate of any population β' value.

.95

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Table 1

Population Intercorrelations[a] Specified in Monte Carlo Procedure and Accompanying Theoretical Standardized Regression Weights

ρ_{y1}	ρ_{y2}	ρ_{12}	β_1	β_2
.95	.95	.95	.4872	.4872
.70	.70	.95	.3590	.3590
.45	.70	.95	-2.2051	2.7949
.70	.45	.95	2.7949	-2.2051
.45	.45	.95	.2308	.2308
.70	.95	.70	.0606	.9020
.45	.95	.70	-.4216	1.2451
.95	.70	.70	.9020	.0686
.70	.70	.70	.4118	.4118
.45	.70	.70	-.0784	.7549
.00	.70	.70	-.9608	1.3725
.95	.45	.70	1.2451	-.4216
.70	.45	.70	.7549	-.0784
.45	.45	.70	.2647	.2647
.00	.45	.70	-.6176	.8824
.70	.00	.70	1.3725	-.9608
.45	.00	.70	.8824	-.6176
-.45	.00	.70	-.8824	.6176
-.70	.00	.70	-1.3725	.9608
.70	.95	.45	.3417	.7962
.45	.95	.45	.0282	.9373
.95	.70	.45	.7962	.3420
.70	.70	.45	.4828	.4828
.45	.70	.45	.1693	.6238
.00	.70	.45	-.3950	.8777
.95	.45	.45	.9373	.0282
.70	.45	.45	.6238	.1693
.45	.45	.45	.3103	.3103
.00	.45	.45	-.2539	.5643
-.45	.45	.45	-.8182	.8182
.70	.00	.45	.8777	-.3950
.45	.00	.45	.5643	-.2539
-.45	.00	.45	-.5643	.2539
-.70	.00	.45	-.8777	.3950

[a] ρ_{y1} is the population correlation between the criterion variable, z_y, and the predictor variable, z_1. ρ_{y2} is the population correlation between the criterion variable, z_y, and the predictor variable, z_2. ρ_{12} is the population correlation between the independent variables, z_1 and z_2. These population correlations were utilized in the determination of factor structure matrices for input into the Monte Carlo technique. There are five factor structure matrices which have a ρ_{12} value of .95, fourteen which have a ρ_{12} value of .70 and fifteen which have a ρ_{12} value of .45.

Table 2

Calculations of the Bias[a] Present in Estimating β_1'

Pop. Correlation Bet. Ind. Variables		Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
$\rho_{12} = .95$							
	Mean	-.004	.010	-.003	-.003	-.009	-.015
	Standard Deviation	.026	.025	.028	.023	.010	.028
	Median	-.001	.002	-.007	-.004	-.009	-.017
	Mode	.001	-.004	.002	.001	-.010	-.002
	Skewness	-.524	1.090	.619	.357	-.098	.224
	Minimum	-.044	-.010	-.032	-.032	-.024	-.050
	Maximum	.027	.051	.039	.032	.004	.024
$\rho_{12} = .70$							
	Mean	-.000	-.002	-.004	-.003	-.005	-.002
	Standard Deviation	.010	.011	.013	.012	.012	.011
	Median	.002	-.002	-.005	-.002	-.006	-.001
	Mode	.015	.019	.022	.005	.013	.006
	Skewness	-.556	.081	.180	-.505	-.851	-.258
	Minimum	-.023	-.024	-.026	-.029	-.035	-.022
	Maximum	.015	.019	.022	.016	.013	.019
$\rho_{12} = .45$							
	Mean	-.004	-.001	.001	-.003	-.000	.001
	Standard Deviation	.007	.013	.011	.008	.013	.008
	Median	-.003	.003	.001	-.000	.001	.001
	Mode	-.003	.003	-.004	.001	.000	.001
	Skewness	-.390	-.806	.152	-1.748	-.004	1.568
	Minimum	-.017	-.031	-.020	-.025	-.019	-.012
	Maximum	.006	.018	.020	.005	.018	.027
	N =		(25)			(100)	

[a]Bias was determined by the following formula: $(\bar{b}_1' - \beta_1')$ where $\bar{b}_1'$ is the mean of the sampling distribution of five-hundred standardized regression weights and β_1' is the corresponding theoretical standardized regression weight.

Table 3

Calculations of the Bias[a] Present in Estimating β_2'

Pop. Correlation Bet. Ind. Variables	Normal	X_5^2	X_{20}^2	Normal	X_5^2	X_{20}^2
ρ_{12} = .95						
Mean	-.004	-.020	-.010	-.003	-.004	.006
Standard Deviation	.025	.019	.029	.029	.013	.026
Median	-.003	-.016	-.005	.002	-.005	.008
Mode	-.001	-.003	-.005	-.002	.007	.001
Skewness	-.029	-1.120	-.726	-.714	.126	-.346
Minimum	-.039	-.052	-.056	-.049	-.017	-.033
Maximum	.030	-.003	.024	.030	.011	.040
ρ_{12} = .70						
Mean	-.003	-.005	-.001	-.002	-.000	-.003
Standard Deviation	.015	.015	.012	.008	.104	.013
Median	-.003	-.004	-.002	-.001	-.003	-.003
Mode	.004	.000	-.020	-.010	.003	-.004
Skewness	.254	.476	.764	.392	.748	-.468
Minimum	-.029	-.028	-.020	-.013	-.017	-.032
Maximum	.026	.027	.028	.014	.029	.020
ρ_{12} = .45						
Mean	-.004	-.005	-.002	-.001	-.004	-.002
Standard Deviation	.007	.008	.008	.006	.009	.007
Median	-.005	-.006	-.002	-.001	-.001	-.001
Mode	-.002	-.006	-.001	-.002	-.005	-.002
Skewness	1.853	.059	.247	-.072	-.773	-.677
Minimum	-.011	-.018	-.016	-.011	-.022	-.019
Maximum	.019	.010	.015	.010	.008	.009
N =		(25)			(100)	

[a] Bias was determined by the following formula: $(\bar{b}_2' - \beta_2')$ where $\bar{b}_2'$ is the mean of the sampling distribution of five-hundred standardized weights and β_2' is the corresponding theoretical standardized weight.

Table 4

Standard Deviation of the Empirical Sampling Distribution of b_1' [a]

Pop. Correlation Bet. Ind. Variables	Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
$\rho_{12} = .95$						
Mean	.428	.431	.445	.417	.436	.420
Standard Deviation	.163	.138	.159	.147	.141	.146
Median	.440	.452	.457	.437	.450	.445
Mode	.179	.214	.202	.187	.214	.184
Skewness	-.483	-.662	-.435	-.593	-.624	-.820
Minimum	.179	.214	.202	.187	.214	.184
Maximum	.623	.588	.643	.589	.600	.570
$\rho_{12} = .70$						
Mean	.175	.195	.182	.177	.194	.184
Standard Deviation	.062	.073	.067	.064	.073	.068
Median	.202	.222	.216	.202	.221	.207
Mode	.215	.260	.221	.213	.238	.225
Skewness	-.731	-.729	-.800	-.629	-.753	-.677
Minimum	.073	.079	.077	.078	.078	.080
Maximum	.256	.265	.252	.258	.273	.272
$\rho_{12} = .45$						
Mean	.135	.150	.137	.134	.151	.139
Standard Deviation	.055	.064	.055	.056	.063	.060
Median	.144	.159	.150	.145	.161	.145
Mode	.078	.079	.081	.077	.077	.072
Skewness	-.405	-.233	-.436	-.345	-.437	-.327
Minimum	.043	.049	.045	.043	.049	.043
Maximum	.212	.248	.214	.221	.226	.219
N =		(25)			(100)	

[a] Where b_1' is a standardized regression coefficient corresponding to z_1 in the prediction equation.

Table 5

Standard Deviation of the Empirical Sampling Distribution of b_2' [a]

Pop. Correlation Bet. Ind. Variables	Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
ρ_{12} = .95						
Mean	.428	.435	.449	.414	.432	.420
Standard Deviation	.164	.143	.163	.145	.141	.147
Median	.441	.453	.459	.437	.448	.446
Mode	.179	.211	.202	.188	.212	.182
Skewness	-.445	-.629	-.409	-.623	-.605	-.851
Minimum	.179	.211	.202	.188	.212	.182
Maximum	.628	.600	.653	.582	.597	.567
ρ_{12} = .70						
Mean	.175	.197	.183	.179	.197	.185
Standard Deviation	.063	.075	.069	.064	.076	.069
Median	.197	.215	.207	.206	.224	.203
Mode	.236	.285	.244	.235	.251	.248
Skewness	-.672	-.559	-.681	-.692	-.727	-.510
Minimum	.074	.081	.070	.075	.074	.077
Maximum	.256	.285	.264	.260	.276	.289
ρ_{12} = .45						
Mean	.136	.155	.142	.137	.153	.141
Standard Deviation	.055	.067	.059	.056	.065	.059
Median	.154	.173	.157	.149	.171	.155
Mode	.047	.048	.047	.047	.046	.043
Skewness	-.490	-.406	-.508	-.453	-.512	-.459
Minimum	.047	.048	.047	.047	.046	.043
Maximum	.206	.249	.214	.210	.234	.219
N =		(25)			(100)	

[a] Where b_2' is a standardized regression coefficient corresponding to z_2 in the prediction equation.

Table 6

Calculations of the Difference[a] Between Empirical Type II Error and Theoretical Type II Error in the Testing of the Hypothesis[b], $H_0:\beta_1' = 0$

Pop. Correlation Bet. Ind. Variables	Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
$\rho_{12} = .90$						
Mean	.211	.220	.218	.124	.136	.126
Standard Deviation	.240	.245	.241	.150	.149	.152
Median	.173	.184	.186	.089	.113	.093
Mode	.000	.000	.000	.000	.000	.000
Skewness	.300	.273	.254	.434	.304	.370
Minimum	.000	.000	.000	.000	.000	.000
Maximum	.479	.502	.485	.320	.329	.303
$\rho_{12} = .70$						
Mean	.053	.053	.058	.016	.016	.014
Standard Deviation	.060	.058	.065	.078	.093	.072
Median	.046	.051	.053	-.005	.002	-.002
Mode	.000	.000	.000	.000	.000	.000
Skewness	.728	.841	.718	.876	-.260	.301
Minimum	.000	.000	.000	-.127	-.209	-.139
Maximum	.165	.181	.193	.211	.199	.157
$\rho_{12} = .45$						
Mean	.012	.023	.019	.019	.022	.023
Standard Deviation	.030	.037	.038	.085	.101	.097
Median	.004	006	.008	.013	.005	.007
Mode	.000	.000	.000	.000	.000	.000
Skewness	.364	1.463	.664	-1.641	-1.280	-.965
Minimum	-.051	-.021	-.057	-.238	-.268	-.247
Maximum	.080	.124	.102	.140	.188	.192
N =		(25)			(100)	

[a]The differences were determined by subtracting the theoretical probability of committing a type II error from the resulting empirical proportions of type II errors committed. Theoretical probabilities were calculated under the assumption of normality in the sampling distribution of standardized regression coefficients.

[b]The hypothesis, $H_0:\beta_1' = 0$ involved an F test at $\alpha = .05$.

Table 6a

Calculations of the Difference[a] Between Empirical Type II Error and Theoretical Type II Error in the Testing of the Hypothesis[b], $H_0: \beta_2' = 0$

Pop. Correlation Bet. Ind. Variables	Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
$\rho_{12} = .90$						
Mean	.213	.213	.212	.124	.122	.135
Standard Deviation	.238	.237	.233	.154	.143	.173
Median	.180	.180	.180	.084	.090	.088
Mode	.000	.000	.000	.000	.000	.000
Skewness	.270	.265	.251	.478	.401	.444
Minimum	.000	.000	.000	.000	.000	.000
Maximum	.475	.481	.481	.331	.307	.355
$\rho_{12} = .70$						
Mean	.060	.067	.067	.020	.015	.019
Standard Deviation	.080	.076	.081	.066	.082	.064
Median	.028	.044	.043	-.003	.000	-.000
Mode	.000	.000	.000	.000	.000	.000
Skewness	.691	.692	.914	1.168	-.562	.726
Minimum	.000	.000	.000	-.098	-.187	-.097
Maximum	.199	.205	.231	.195	.161	.167
$\rho_{12} = .45$						
Mean	.029	.027	.032	.031	.038	.032
Standard Deviation	.054	.045	.050	.097	.107	.105
Median	.013	.012	.007	.018	.028	.014
Mode	.000	.000	.000	.000	.000	.000
Skewness	.514	1.846	1.377	-1.025	-1.451	-1.123
Minimum	-.065	-.017	-.025	-.238	-.272	-.264
Maximum	.142	.140	.162	.170	.172	.184
N =		(25)			(100)	

[a]The differences were determined by subtracting the theoretical probability of committing a type II error from the resulting empirical proportions of type II errors committed. Theoretical probabilities were calculated under the assumption of normality in the sampling distribution of standardized regression coefficients.

[b]The hypothesis, $H_0: \beta_2' = 0$ involved an F test at $\alpha = .05$.

Table 7

Proportion of False Hypotheses Retained in Testing $H_0:\beta_1' = 0$

Pop. Correlation Bet. Ind. Variables	Normal	χ^2_5	χ^2_{20}	Normal	χ^2_5	χ^2_{20}
$\rho_{12} = .95$						
Mean	.422	.431	.429	.238	.249	.239
Standard Deviation	.464	.468	.465	.303	.300	.310
Median	.364	375	.377	.156	.181	.149
Mode	.000	.000	.000	.000	.000	.000
Skewness	.242	.225	.215	.432	.392	.480
Minimum	.000	.000	.000	.000	.000	.000
Maximum	.934	.958	.940	.616	.620	.646
$\rho_{12} = .70$						
Mean	.259	.259	.264	.125	.125	.123
Standard Deviation	.362	.355	.360	.215	.202	.209
Median	.082	.089	.090	.004	.015	.007
Mode	.000	.000	.000	.000	.000	.000
Skewness	1.042	1.016	.987	1.343	1.258	1.341
Minimum	.000	.000	.000	.000	.000	.000
Maximum	.926	.890	.912	.574	.510	.562
$\rho_{12} - .45$						
Mean	.277	.288	.284	.122	.126	.127
Standard Deviation	.342	.335	.347	.197	.185	.196
Median	.132	.176	.139	.024	.032	.026
Mode	.000	.000	.000	.000	.000	.000
Skewness	.880	.810	.856	1.500	1.348	1.358
Minimum	.000	.000	.000	.000	.000	.000
Maximum	.940	.928	.926	.614	.584	.606
N =		(25)			(100)	

Table 7a

Proportion of False Hypotheses Retained in Testing $H_0:\beta_2' = 0$

Pop. Correlation Bet. Ind. Variables		Normal	χ_5^2	χ_{20}^2	Normal	χ_5^2	χ_5^2
$\rho_{12} = .95$							
	Mean	.424	.424	.423	.238	.235	.248
	Standard Deviation	.462	.461	.457	.306	.298	.328
	Median	.371	.371	.371	.155	.156	.154
	Mode	.000	.000	.000	.000	.000	.000
	Skewness	.224	.221	.211	.427	.436	.443
	Minimum	.000	.000	.000	.000	.000	.000
	Maximum	.930	.936	.936	.614	.610	.658
$\rho_{12} = .70$							
	Mean	.277	.279	.278	.129	.125	.129
	Standard Deviation	.363	.350	.359	.221	.198	.216
	Median	.109	.111	.081	.008	.017	.010
	Mode	.000	.000	.000	.000	.000	.000
	Skewness	.925	.863	.917	1.403	1.302	1.397
	Minimum	.000	.000	.000	.000	.000	.000
	Maximum	.952	.880	.942	.612	.514	.604
$\rho_{12} = .45$							
	Mean	.336	.334	.339	.148	.155	.149
	Standard Deviation	.364	.334	.352	.206	.200	.201
	Median	.220	.241	.244	.035	.061	.040
	Mode	.000	.000	.000	.000	.000	.000
	Skewness	.521	.523	.482	1.071	.970	.983
	Minimum	.000	.000	.000	.000	.000	.000
	Maximum	.932	.944	.932	.614	.580	.588
	N =		(25)			(100)	

Table 8

Spearman Correlation Coefficients[a]

	Dist. Type[c]	ρ_{12}[d]		Dist. Type	ρ_{12}
$Bias_1$[b]	-.02 p<.43	-.02 p<.41	$Bias_1$	.04 p<.34	-.19 *p<.03
$Bias_2$	.05 p<.31	-.10 p<.16	$Bias_2$	-.00 p<.49	.04 p<.36
$Diff_1$ (25)[e]	.05 p<.31	.28 *p<.00	$Diff_1$ (100)	-.00 p<.48	.06 p<.26
$Diff_2$ (25)	.02 p<.41	.24 *p<.01	$Diff_2$ (100)	.00 p<.50	-.00 p<.49
S_{e_1}[f]	.05 p<.32	.60 *p<.00	S_{e_1}	.03 p<.38	.59 *p<.00
S_{e_2}	.06 p<.29	.59 *p<.00	S_{e_2}	.02 p<.41	.58 *p<.00
N =	(25)			(100)	

[a]Some of the correlation coefficients tabled were calculated on variables whose elements involve statistics of sampling distributions. These statistics were tabulated from regression equations originally involving a sample size of 25 or a sample size of 100. The number of cases upon which the significance was determined was 102: the number of factor structures (34) multiplied by the number of distributions sampled (3), which equals the number of sampling distributions examined.

[b]See notes tables 2 and 3.

[c]Dist. Type refers to the shape of the population from which the z scores were generated for input into the regression equations for the purpose of constructing sampling distributions. Three distributions were involved: normal, χ^2_5 and χ^2_{20}.

[d]ρ_{12} is the population correlation between the predictor variables. Three levels were examined: .95, .70 and .45.

[e]$Diff_1$ (25) can vary between zero and one and was calculated by subtracting the theoretical probability of committing a type II error from the empirical proportion of type II errors committed in the testing of the hypothesis $H_0:\beta_1' = 0$, at $\alpha = .05$. $Diff_2$ (25) was determined in the same manner for the hypothesis $H_0:\beta_2' = 0$, as was $Diff_1(100)$ and $Diff_2(100)$.

[f]S_{e_1}, is the empirical standard deviation of the sampling distribution of b_1 values.

*Significant at $\alpha = .05$.

Table 9

Pearson Correlation Coefficients[a]

	$Bias_1$	$Bias_2$	$Diff_1$ (25)[e]	$Diff_2$ (25)	S_{e_1}[f]	S_{e_2}
$Bias_1$	1.00	-.65 *p .00	.13 p .19	.20 *p .05	.07 p<.49	.08 p<.40
$Bias_2$	-.65 *p<.00	1.00	-.30 *p<.00	-.29 *p<.00	-.29 *p<.00	-.28 *p<.01
$Diff_1$ (25)	.13 p<.19	-.30 *p<.00	1.00	.86 *p<.00	.72 *p<.00	.70 *p<.00
$Diff_2$ (25)	.20 *p<.05	-.29 *p<.00	.86 *p<.00	1.00	.66 *p<.00	.70 *p<.00
S_{e_1}	.07 p<.49	-.29 *p<.00	.72 *p<.00	.66 *p<.00	1.00	.99 *p<.00
S_{e_2}	.08 p<.40	-.28 *p<.01	.70 *p.<00	.70 *p<.00	.99 *p<.00	1.00
N -			(25)			

(See notes table 8)

*Significant at $\alpha = .05$.

Table 10

Pearson Correlation Coefficients[a]

	$Bias_1$[b]	$Bias_2$	$Diff_1$ (100)[e]	$Diff_2$ (100)	S_{e_1}[f]	S_{e_2}
$Bias_1$	1.00	-.65 *p<.00	-.12 p<.21	-.06 p<.56	-.22 *p<.03	-.22 *p<.02
$Bias_2$	-.65 *p<.00	1.00	-.06 p<.57	-.10 p<.34	-.06 p<.57	-.02 p<.81
$Diff_1$ (100)	-.12 p<.21	-.06 p<.57	1.00	.65 *p<.00	.57 *p<.00	.58 *p<.00
$Diff_2$ (100)	-.06 p<.56	-.10 p<.34	.65 *p<.00	1.00	.59 *p<.00	.57 *p<.00
S_{e_1}	-.22 *p<.03	-.06 p<.57	.57 *p<.00	.59 *p<.00	1.00	.99 *p<.00
S_{e_2}	-.22 *p<.02	-.02 p<.81	.58 *p<.00	.57 *p<.00	.99 *p<.00	1.00
N =			(100)			

(See notes table 8)

*Significant at $\alpha = .05$.